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THE ASTRONOMICAL WORK OF CARL FRIEDRICH GAUSS (1777-1855)

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This paper was presented on 3 June 1977 at the Royal Society of Canada's Gauss Symposium at the Ontario Science Centre in Toronto [1].

SUMMARIES

Gauss's interest in astronomy dates from his student-days in Göttingen, and was stimulated by his reading of Franz Xavier von Zach's Monatliche Correspondenz... *where he first read about Giuseppe Piazzi's discovery of the minor planet Ceres on 1 January 1801. He quickly produced a theory of orbital motion which enabled that faint star-like object to be rediscovered by von Zach and others after it emerged from the rays of the Sun. Von Zach continued to supply him with the observations of contemporary European astronomers from which he was able to improve his theory to such an extent that he could detect the effects of planetary perturbations in distorting the orbit from an elliptical form. To cope with the complexities which these introduced into the calculations of Ceres and more especially the other minor planet Pallas, discovered by Wilhelm Olbers in 1802, Gauss developed a new and more rigorous numerical approach by making use of his mathematical theory of interpolation and his method of least-squares analysis, which was embodied in his famous* Theoria motus *of 1809. His laborious researches on the theory of Pallas's motion, in which he enlisted the help of several former students, provided the framework of a new mathematical formulation of the problem whose solution can now be easily effected thanks to modern computational techniques.*

Up to the time of his appointment as Director of the Göttingen Observatory in 1807, Gauss had little opportunity for engaging himself in practical astronomical work. His first systematic observations were concerned with re-establishing the latitude of

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of that observatory, which had been well-determined by Tobias Mayer more than fifty years earlier. However, he found a small but not negligible discrepancy between results obtained independently from stellar and solar observations, as well as irregularities among later measurements of polar altitudes (made at the new observatory completed in 1816), which he was never able to explain, despite repeated attempts to do so using different instruments and observational techniques. Similar anomalies were also detected by a number of other astronomers at around this time. These may have been associated--at any rate, partially--with the phenomenon identified later in the century as a "variation of latitude" due to minor periodic fluctuations in the Earth's axis of rotation produced by meteorological and geological factors.

L'intérêt de Gauss pour l'astronomie remonte à l'époque de ses études à Göttingen et fût stimulé par sa lecture du Monatliche Correspondenz... *de Franz Xavier von Zach où il recueillit ses premières informations sur la découverte, le 1ier janvier 1801, de l'astéroide Céres par Giuseppe Piazzi. Peu après, il édifia une théorie des mouvements orbitaux qui permit à von Zach et à d'autres de redécouvrir cette simili-étoile de faible intensité après sa ré-émergence du Soleil. Von Zach continua à lui fournir les observations des astronomes européens de cette époque ce qui permit à Gauss d'améliorer sa théorie à un point tel qu'il put déceler les effets des perturbations planétaires qui écartent les orbites de leur forme elliptique. Pour pallier à la complexité ainsi introduite dans les calculs relatifs à Céres et, plus particulièrement, à l'astéroide Pallas, découverte par Wilhelm Olbers en 1802, Gauss développa une nouvelle méthode d'approximation plus rigoureuse en employant sa théorie mathématique de l'interpolation et sa méthode des moindres carrés, qui fut incorporée à son célèbre* Theoria motus *de 1809. Ses intenses recherches sur la théorie du mouvement de Pallas, dans lesquelles il impliqua plusieurs anciens étudiants, fournirent le cadre d'une nouvelle formulation mathématique du problème dont la solution peut maintenant être facilement réalisée grâce aux techniques modernes de calcul.*

Jusqu'au moment de sa nomination au poste de Directeur de l'Observatoire de Göttingen, en 1807, Gauss eut rarement l'occasion de s'engager dans des travaux d'astronomie pratique. Ses premières *observations systématiques furent reliées à la redétermination de la latitude de l'Observatoire que Tobias Mayer avait bien calculé 50 ans auparavant. Cependant, il trouva des différences, petites quoique non négligeables, entre les résultats issues indépendamment d'observations stellaires et solaires, aussi bien que des irrégularités dans les mesures ultérieures des altitudes polaires (complétées au nouvel observatoire en 1816) qu'il fut toujours incapable d'expliquer, malgré des essais répetés en employant divers instruments et diverses techniques d'observation. A peu près à cette même époque, des anomalies similaires furent aussi relevées par nombre d'autres astronomes. Ceci peut avoir été en relation, --à tout le moins partiellement--avec le phénomène identifié plus tard dans le siècle comme la "variation de la latitude", découlant de fluctuations périodiques mineures de l'axe de rotation de la Terre, fluctuations elles-même produites par facteurs météorologiques et géologiques.*

The most useful and important contribution to our knowledge of Carl Friedrich Gauss's astronomy is without doubt the 258-page monograph by Professor Martin Brendel (1929) published in Volume XI_2 of Gauss's *Werke*. Brendel there presents a synthetic view of Gauss's researches in this field structured into two parts dealing first with practical and spherical astronomy, then with various aspects of theoretical astronomy. The primary sources used include Gauss's own German and Latin treatises, as well as his contributions to Franz Xavier von Zach's *Monatliche Correspondenz zur Beförderung der Erd-und Himmels-Kunde* and the *Göttingen Anzeigen von gelehrten Sachen*. The Gauss manuscripts in the Göttingen University library for which Brendel prepared his own detailed index, include numerous sets of calculations and astronomical tables but, since there are few words of explanation or graphical illustrations, they are very difficult even to classify let alone interpret. Much more information about Gauss's methods, instruments, theoretical insights, etc. is therefore to be found in his voluminous correspondence with eminent contemporary astronomers such as Wilhelm Olbers (1758-1840), Franz von Zach (1754-1832) and Friedrich Wilhelm Bessel (1784-1846); also, with talented pupils like Friedrich Bernhard Gottfried Nicolai (1793-1846), Johann Franz Encke (1791-1865), Bernhard August von Lindenau (1779-1854), Heinrich Christian Schumacher (1780-1850), and Christian Ludwig Gerling (1788-1864). Extracts from letters to and from Gauss are printed not only in Brendel's memoir but throughout the whole of the standard twelve-volume edition of Gauss's *Werke*.

Yet, despite the extensiveness of these sources, and the care with which he examined and collated them, Brendel remarks in his memoir that he felt unable to explain why Gauss did what he did (in astronomy) when he did. To my knowledge, this situation has not changed during the intervening four decades. Perhaps the continued efforts of the *Gauss Gesellschaft* to publish and publicize additional facts and documents pertaining to Gauss's life and work will, in time, furnish some fresh clues to illuminate the picture which already exists of Gauss's rôle in local, national, and international scientific developments during the first half of the nineteenth century.

The more modest aim of this paper is to present a modern reappraisal of Gauss's astronomical work on the basis of the same historical evidence cited above. Contrary to Brendel, however, I prefer to begin by considering Gauss's contributions to *theoretical* astronomy and then to *practical* astronomy, since this follows the historical sequence of events. Broadly speaking, Gauss's researches from 1801-1818 were centred on mathematics and the computation of orbits; whereas, from 1818 onwards, they were concerned primarily with observational astronomy, geology, and geomagnetism.

There can be little doubt that Gauss's interest in astronomy was aroused by the books he read and the lectures he attended while a student in Göttingen from 1795-1798. He also appears to have done a little casual observing during that time. Not long after he returned to his home-town of Brunswick to lecture on mathematics at the Collegium Carolinum, he assisted an army officer, Lieut. K. L. E. von Lecoq, who had been ordered to make military maps of Westphalia, with astronomical calculations associated with this project. It was Lecoq who first encouraged Gauss to make systematic astronomical observations, and to seek advice and assistance from Franz von Zach, the Director of the Seeberg Observatory near Gotha. Von Zach's response to Gauss's written request to visit his observatory was not encouraging, apparently because the Duke of Saxe-Gotha had already been objecting to visitors misusing his astronomical instruments. Nevertheless, a contact was thus established between the young mathematician and the mature astronomer which was to prove of great benefit to both as well as a source of encouragement to many other eminent contemporaries involved in studies of this nature.

The first sign of this mutual co-operation was the publication in Zach's journal of a paper by Gauss on the calculation of the date of Easter (1800). It has been suggested that the motivation behind this publication was simply the desire on its author's part to develop a rigorous numerical method which would establish, among other things, his own date of birth; for all that his mother had been able to tell him was that he had been born on the Wednesday before Rogation Sunday in 1777. Ironically, the original record of that event in the register of St. Catherine's Church in Brunswick bore no date! Only later, probably much later, were the words "on the 30th April" added to the official entry. Personally, I think that a more probable stimulus was of a literary character; and that Gauss's thoughts on this subject were roused after reading an article on this subject by Johann Heinrich Lambert in the Berlin *Astronomisches Jahrbuch* for 1778. The fact that rough notes written in Gauss's own hand and dated 1798 appear on the fly-leaves of his personal copy of a collection of astronomical tables published under the auspices of the Prussian Academy of Sciences in 1776, indicates that a link already existed before the year 1800 between his arithmetical researches and astronomical studies.

The circumstance which was destined to determine the initial course of Gauss's scientific career was his successful attempt to solve a problem which presented itself to him during a perusal of a report in von Zach's journal for September 1801, concerning Giuseppe Piazzi's chance discovery of a new planet (Ceres) nine months earlier. This discovery had provoked great excitement among astronomers, since they had long been aware of a gap in an arithmetical series for predicting planetary distances known as the Titius-Bode Law, which suggested the existence of a "missing" planet orbiting between Mars and Jupiter. There appeared to be no physical justification for this series, yet it had been formulated *before* the discovery of Uranus by William Herschel in 1781 and subsequently found to yield a satisfactory prediction of that planet's distance from the Sun. It was easy to understand why Ceres had hitherto escaped notice; it was so small that it reflected less light than any of those other planets and could consequently be seen only as a very faint object through a telescope.

The practical difficulty which arose during 1801 was that before astronomers had received news of Piazzi's discovery and had time to observe this interesting new celestial object for themselves, its motion had carried it into a region of the sky close to the Sun where it was hidden by the Sun's rays. It was not expected to be visible again before December and no-one knew exactly how fast or in precisely what direction it was travelling. Where should one point a telescope in order to re-discover it after so many months had passed? There are thousands of stars of greater or equal brightness and there was nothing unusual in its appearance to enable it to be distinguished from a faint (eighth magnitude) star. The theoretical difficulty was that the time-interval of 41 days between the first and last of Piazzi's observations corresponded to an arc of only 9° in the sky, from which it was scarcely possible to estimate the curvature hence the size and shape, of its orbit. This relatively slow rate of motion also implied that it would have to be viewed over a period of several hours before its movement--the only feature by which it could be recognised--became noticeable.

One of the first things which Gauss did, therefore, when he decided to tackle the problem of defining Ceres' orbit, was to develop a fundamental formula applying only to elliptical planetary orbits, and use it to calculate Ceres' distance *from the Earth* at three of the times when it had been observed by Piazzi. In order to do so, he had to know its observed positions at those times and to assign values to the size and shape of its orbit as well as its position in relation to the perihelion, or point of nearest approach to the Sun. This part of his procedure involved a number of trial assumptions until a satisfactory internal agreement was established between theory and observation. What Gauss wanted to know, however, were Ceres' distances *from the Sun* at those three observed times. The second part of his procedure was therefore concerned with transferring the system of reference from the Earth to the Sun. This introduced a further three parameters. Thus Gauss worked from the assumption that Ceres' orbit was elliptical with the Sun at a focus, and used his mathematical skill to calculate six theoretical quantities, or elements. These, in effect, replaced the three observed positions from which they had been derived. However, they had a more general significance; for they uniquely specified the size, shape, and orientation of the orbit in space, from which the celestial position of Ceres in it could be calculated *at any past or future time*.

The elements used by Gauss to calculate the ephemeris for Ceres which von Zach published in the December 1801 issue of his journal were not the first, but the fourth set obtained after taking account of improved data from Piazzi and removing inconsistencies that he had found when comparing von Zach's and Piazzi's respective solar tables. The importance of Gauss's calculations may be appreciated from the fact that they placed Ceres several degrees away from independent predictions by Wilhelm Olbers and Giuseppe Piazzi based upon the simpler assumption of a circular orbit, and from others by Johann Franz Burckhardt also based on an elliptical orbit but using a totally different method. In general, German astronomers preferred to place their faith in their young fellow-countryman, and were successful in their search for Ceres in the region of the sky where he had predicted it should be. Von Zach happened to be the first to rediscover it on 1 January 1802--exactly one year after Piazzi's original discovery. French astronomers at first followed Burckhardt's predictions before turning to Gauss's, and consequently took longer to observe Ceres. When they did, however, they were quick to acknowledge Gauss's achievement. Gauss himself openly declared his indebtedness to von Zach for the speed with which news of Piazzi's discovery and the necessary data had been disseminated.

The only way in which a knowledge of Ceres' orbit could be further improved was by gathering more and better data extending over a longer interval of time and hence over a still wider arc of its orbit. It was this consideration which inspired Gauss, Olbers, von Zach, and others to examine old star-catalogues and star-charts in the hope of finding Ceres wrongly listed as a star in a place where no star was any longer visible. A similar search three decades earlier had met with success when Uranus was found listed as star No. 964 in the catalogue of the Göttingen astronomer Tobias Mayer. Von Zach therefore suggested this same catalogue as a potential source for a pre-discovery observation of Ceres; others were the catalogues of John Flamsteed, James Bradley, Nicolas de Lacaille, and Jerôme de Lalande. However, it soon transpired that all the uncertainties could be accounted for by the phenomenon of variations in brightness, misrecordings or misprints in those catalogues, or errors in the reductions of the raw data. This work nevertheless served to emphasise the need for a thorough re-editing of those older catalogues introducing more precise corrections for the effects of astronomical refraction, stellar aberration, and the precession of the equinoxes.

A related problem was that of ensuring a high degree of accuracy in the positions of stars to which Ceres' own positions had to be referred. The importance of this requirement had been fully appreciated by Mayer some fifty years previously when developing his lunar theory; now Olbers and Friedrich Wilhelm Bessel began to pay particular attention to it. Olbers proposed to von Zach that it would greatly assist astronomers if charts showing Ceres' predicted future motion against a background of stars were to appear from time to time in his journal. The usefulness of such a visual aid became all the more apparent when, only a few days later, Olbers made a chance discovery of a second minor planet (Pallas); and after a third (Juno) was discovered in September 1804 by the Göttingen astronomer Karl Ludwig Harding. It was Harding himself who had meanwhile begun to undertake this work. No fewer than seven of his charts--two for Ceres, three for Pallas, and two for Juno--were published by von Zach between March 1803 and April 1807. All of the orbits drawn by Harding were based upon Gauss's current ephemerides. The charts themselves did much to stimulate the interest of many European astronomers in this new field of positional astronomy: they appear to have motivated Gauss's calculations of the boundaries in celestial latitude beyond which Ceres and Pallas could never move, and of the times of year when these limits would be attained (1804).

By now, Gauss was aware of the fact that his assumption that these celestial bodies move in elliptical orbits was not strictly true. This was revealed by the distribution of the errors between the new observational data being steadily accumulated, and by the theoretical predictions based on the best elements that he was able to compute from those data. He rightly interpreted these perturbations as being due to the gravitational attraction of other planets--particularly Jupiter, the nearest and most massive--pulling Ceres and Pallas out of their otherwise elliptical paths. His first efforts to develop a new theoretical approach, which made use of some of his early mathematical researches, had to be abandoned because it gave rise to an impossibly large computational burden. An alternative involving interpolation of the perturbation function, which he developed in 1805, proved to be more tractable and became the foundation of his logically coherent and mathematically elegant theory of the motion of the celestial bodies which was ultimately published in Latin four years later (1809). This theoretical masterpiece includes Gauss's first published account of his method of least-squares analysis which enabled him to make use of *all*--not merely three--observed positions of a planet when deducing the most probable value of the six elements on which his predictions were based. The underlying principle has come to be of enormous importance in astronomy, geodesy, and indeed all branches of applied mathematics and statistics.

After completing that treatise, Gauss continued working on the general theory of Pallas's perturbations and delivered a disquisition on it to the Royal Society of Sciences in Göttingen on 25 November 1810 (1811). This research presented the greater challenge to his mathematical ingenuity because Pallas's orbit was more elliptical than that of Ceres, and inclined at a greater angle to that of Jupiter. An additional stimulus was the Paris Academy of Sciences' offer of a 6000-Franc prize to anyone who could develop such a theory, although this was later to become a secondary consideration to him. It was while he was engaged on this research that he made a discovery for which he wanted to establish a claim to priority without disclosing it, and followed a precedent set by several other famous scientists before his time in deciding to publish it as an anagram:

$$1\ 1\ 1\ 1\ 0\ 0\ 0\ 1\ 0\ 0\ 1\ 0\ 1\ 0\ 0\ 1$$

We can infer from letters written about this time to his friends Bessel and Olbers that this anagram refers to his recognition of the period-relation that 7 revolutions of Jupiter are equal to 18 revolutions of Pallas, about the Sun. This suggests that the anagram should be broken up into a sequence of four numbers expressed in binary notation, thus:

$$111 = 7, \quad 1000 = 8, \quad 10010 = 18, \quad 1001 = 9$$

The question which this raises is: what is the significance of the numbers 8 and 9, in this context? We may presume that they relate in some way to Pallas and to Jupiter. The answer which I prefer, out of several that various authors have proposed [2], is that they refer to the positions of those two planets in order of increasing distance from the Sun, as in the case of Bode's Law. Five years beforehand Olbers had discovered a fourth minor planet (Vesta), and Gauss's calculations had meanwhile established the following progression in the planetary distances up to Jupiter:

1	2	3	4	5	6	7	8	9
Mercury	Venus	Earth	Mars	Vesta	Juno	Ceres	Pallas	Jupiter

Thus the meaning of Gauss's anagram is easily inferred. It should, at the same time, be stated that Bode's Law was not contradicted by the interposition of three minor planets in addition to Ceres, since Olbers's discovery of Vesta had verified his hypothesis that these should all be regarded as fragments of a single major planet which, at some time in the remote past had become disrupted (most probably by gravitational forces).

Such was the labour involved in his investigation of the perturbations of Jupiter on Pallas, that Gauss began to enlist the help of some of his best pupils to make analogous calculations using his method. To Friedrich Bernhard Gottfried Nicolai, by now an assistant at the Mannheim Observatory, he assigned the task of finding the perturbations of Saturn on Pallas. Johann Franz Encke agreed to do the same for Mars. A comparison of their independent results with those of Gauss served to show that Jupiter's mass had previously been underestimated by about 2%. After the completion of this work, Gauss continued to mete out similar computational tasks on minor planets' and comets' orbits to his abler students. The major assignment, which was tackled primarily be Johann Heinrich Westphal, involved producing an auxiliary table from a total of roughly half-a-million figures! Gauss's own work on Jupiter's perturbational effects was so tedious that he did not succeed in completing it before the extended time-limit for the submission of prize-essays, set by the Paris Academy as 1st October 1816; thus he failed to obtain any financial reward for those labours. Thanks to Encke's initiative, Gauss's method was finally made accessible to astronomers all over the world through its publication as a supplement to the Berlin *Astronomischen Jahrbüchern* for 1837 and 1838. It was destined to become the theoretical backbone of numerous profound researches in celestial mechanics which have recently proved to be of great practical importance in this era of space-age technology.

Let us now transfer our attention to the other major aspect of Gauss's astronomy--his observational work at the Göttingen Observatory. Although the climax of this activity was not reached until after he had ceased to be preoccupied with the theory of Pallas's orbit, this does not mean to say that Gauss's interest in observational astronomy was acquired in later life.

On the contrary, ever since beginning his calculations on Ceres' orbit he was anxious to observe the heavens for himself. The failure in 1804 of a project to have a small observatory built for him in Brunswick was principally due to the lack of encouragement by von Zach, on whom he was at that time dependent for well-qualified advice. Von Zach seems to have felt that Gauss would be wasting his talents on routine practical work, although Gauss felt that it was just as important for providing a reliable basis for his theory, as his theory was in accounting for observations. After this disappointment, Gauss began pinning his career hopes on the possibility of a call to Göttingen, to assume the Directorship of the new observatory which had just begun to be constructed to the south-east of what had once been Tobias Mayer's observatory, of which no trace now remains.

This ambition was realised three years later, although he had still to wait almost a decade before the new building was completed. He began to make regular astronomical observations in the *old* observatory towards the end of 1808. In addition to a number of instruments of such minor importance that they need not be specified, he had at his disposal Mayer's 6-foot radius mural quadrant, an accurate pendulum clock made by John Shelton, a large reflecting telescope made by William Herschel, and an achromatic refracting telescope by the London firm of John and Peter Dollond. This last instrument was found by him to be best-suited to minor planet and cometary observations, when used in conjunction with a position-micrometer--an attachment which enabled small angular distances from stars to be measured with a high degree of precision. However, these were merely *relative* observations and no substitute for those made with a meridian transit circle such as many of the best astronomers then possessed, which yielded the *absolute* values of celestial co-ordinates. Thus he resolved to obtain such an instrument from the Hamburg instrument-maker Johann Georg Repsold with whom he was already acquainted. In accordance with Gauss's written instructions, Repsold made extensive modifications to an already-used instrument which he had for sale [3] and eventually brought it with him to Göttingen in 1818, and helped Gauss to instal it in the new observatory which had finally been completed (after numerous delays) two years previously.

The other major additions to his instrumentation were obtained from the newly established Munich-based firm of Reichenbach, Utzschneider, and Liebherr. They included a Repeating Circle--an instrument first invented by Mayer, but by now greatly modified--and a theodolite; also, more importantly, a new meridian circle to replace the Repsold circle, and a transit instrument. Later, Gauss was to acquire a Liebherr-pendulum clock to replace the Shelton regulator for taking the times when the various heavenly bodies crossed the meridian. This was later superceded by an even more reliable regulator by the London clockmaker William Hardy, gifted by Augustus Frederick, Duke of Sussex, the most liberal-minded of George IV's sons.

Now one of the first, and most important tasks of any astronomical observer is to establish the precise latitude of his observatory. Gauss was no exception. The results of his earliest attempts to redetermine the latitude of the old Göttingen Observatory using a sextant borrowed from his colleague Harding, are contained in two publications of 1808 (b and c). He also published, earlier in that same year, general tables of nutation and aberration which are required in the reduction of all astronomical observations (1808a). A few years later, he published other auxiliary tables useful in the case of solar observations (1811, 1812). Gauss's attempt to repeat this latitude determination by coupling his observations of the pole star at the old observatory with a series of solar observations at the new observatory around the times of the summer and winter solstices, led him to detect a small inconsistency; namely, that the latitude deduced from the former was on average about 5" greater than that found from the latter. Initially, he ascribed this to the bending of the viewing telescope and tried to compensate it by placing a weight on that telescope at appropriate places. However, yet another attempt to find his latitude from observations of the pole star, this time using the Repsold Circle at the new observatory (again making due allowance for the mean latitude and longitude differences between the two observatories), yielded the same discrepancy, thereby proving that it could not be due to an instrumental error. Independent confirmation was received from the brother of Heinrich Christian Schumacher, who had also found the same effect in observations of polar altitudes made using a Reichenbach astronomical theodolite. Thus Gauss could not be held responsible either.

The reality of the phenomenon was indisputable. Yet, despite further attempts to explain it, and very numerous and carefully conducted observations of circumpolar stars with his Reichenbach meridian circle in 1819 devoted entirely to the purpose of finding the altitude of the north celestial pole, hence his latitude, Gauss was unable to discover a satisfactory solution. But he now appreciated more than ever the need to subject every major astronomical instrument to very careful investigation for the presence of errors peculiar to itself; for these, if not properly quantified, would falsify celestial positions and theories of motion based on them. This same realisation had likewise dawned upon the English Astronomer Royal John Pond, as a result of his discovery of similar inexplicable anomalies in his own meridian circle observations at Greenwich and those of John Brinkley at the Dunsink Observatory in Ireland. The interpretation of these anomalies and the different results obtained by different observers was soon to become a much debated subject among contemporary German astronomers also [4]. Gauss's own view was that they were, in general, caused by the influence of gravity on the different component parts of each instrument, although his own tests indicated that he had to rule out this possibility as far as his own Reichenbach Circle was concerned.

Gauss's observations from 1820 onwards suffered frequent interruptions, due to his involvement in two other major aspects of related activity--geodesy and geomagnetism--which lie outwith the scope of this paper. There is, however, an intimate connection between the work just described and the geodetic survey that Gauss made in collaboration with Schumacher and others from 1821 to 1824, which must be mentioned. Gauss's astronomical experience had convinced him that little confidence could be placed on the result of a comparison between astronomical observations by himself and others, unless all the data were to be collected with the same instrument. Thus he requested and received permission to transport the Ramsden Zenith sector [5] with which the latitudes of several Danish towns had been made, from Altona to Göttingen where he could use it to determine yet again the latitude of the new Göttingen Observatory. The results for the latitudes of those two observatories were published by him in 1828.

The point which I particularly wish to emphasise, however, is that Gauss had another motive in requesting the zenith sector to prosecute this research; namely, to try to discover the source of the mysterious discrepancy from another standpoint by comparing two sets of identical *stellar* observations made in the *same* observatory with *different* instruments. This is why he now followed the same method with the zenith sector as he had used beforehand with his Reichenbach Circle--determining the zenith point (the point directly above our heads) from observations of the pole star and its image reflected from a mercury horizon, and considering the possibility of the viewing telescope bending under its own weight. This is also why he later continued to take great pains to quantify the instrumental errors of the Reichenbach Circle and to do all that was humanly possible to eliminate them. Although he never published a full account of these researches, he did discuss the dividing errors of his instrument in letters written during 1826 to Bessel, Schumacher, and Olbers, and in the treatise of 1828 alluded to above. Despite his perseverance, however, he came no closer to a solution of his puzzle. Perhaps the lack of harmony among the stellar data collected by himself and by other observers with different instruments in different latitudes, reflects the presence of minute fluctuations in the direction of the Earth's axis of rotation [6]. As a result of a further century-and-a-half of international scientific co-operation we now know that these are produced by a combination of complex geophysical and meteorological phenomena. I am, on the other hand, inclined to believe that the larger discrepancy between the stellar and solar observations, which Gauss himself attributed to equal errors in each of these sets of data [7], may have arisen from the neglect of the Sun's variable and finite distance from the Earth in the method of deducing latitude from the solar observations.

I hope that I have been able to convey a rough picture of what Gauss, in the course of more than half a century of scientific activity, succeeded in contributing to both theoretical and practical astronomy. I have tried to indicate when and why he did what he did, what the objects of his vast computational work were, and why he was later to devote so much of his time to the quantitative assessment of instrumental errors in his equipment. Naturally, I have been unable to incorporate all areas of his astronomical researches, or to portray the full variety of the astronomical phenomena which he and his assistants observed during a life-long association with the Göttingen Observatory. The originality of his pioneering contributions to the development of new mathematical methods of orbital determination will, however, always remain his greatest claim to fame as an astronomer.

NOTES

1. A German version of this paper, based upon the author's Open Lecture to the Astronomische Gesellschaft in Göttingen on 2 March 1877, has already been published in *Sterne und Weltraum* 16 (1977), 158-166.

2. See, for example, Wietzke A 1930 Zur Lösung eines rätselhaften Gauss'schen Anagrammme *Astronomische Nachrichten* 240, 403; MacDonald T L 1931 The Anagram of Gauss *ibid* 241, 31; Benham W 1974 The Gauss Anagram: An Alternative Solution *Annals of Science* 31, 449-55.

3. Details of this instrument, and the modifications made to the mounting, collimation, microscopes, illumination of the cross-wires, graduation of the declination circle, and above all to the design and arrangement of the vernier scales, can be found in an exchange of letters between Gauss and Repsold from 12 January 1815 to 24 January 1818, which comprise the main part of a correspondence published after Brendel's memoir in *Mitteilungen der Mathematischen Gesellschaft in Hamburg* 6 (1929), 398-431.

4. See, for example, Bohnenberger J. G. F. 1817 Zusatz zu dem Schreiben der Herrn Hofrat Gauss *Zeitschrift für Astronomie* 4, 141; and Bessel F. W. 1876, Über die Abweichung der Fixsterne, in Engelman R. (ed.) *Abhandlungen von Friedrich Wilhelm Bessel* 2, 248-51 etc. Leipzig.

5. This was the same instrument with which the British Ordnance Society had been begun in 1787, and linked with the French trigonometrical survey. It is described in Mudge W and Dalby I 1799, An Account of the operations carried on for accomplishing a Trigonometrical Survey of England and Wales, 1. London; from which a French extract was published in Bigourdan G 1912 *Grandeur et Figure de la Terre* Paris, 353-6.

6. This phenomenon is known as "the variation of latitude". Up to 1890, its periodicity was not known. Then S. C. Chandler showed that the Earth's north and south poles wander round within a circle of about 50 ft. in diameter in about 428 days. This was found to be consistent with the Earth's having a small elasticity less than that of steel. Chandler subsequently recognised the existence of two variable angular components in stellar co-ordinates with periods of about 430 days and one year and amplitudes of the order of 0".1 in each case. The melting of the polar ice-caps and oceanic currents are among the causes of these irregularities. Perhaps the near conjunction of Jupiter, Saturn, Uranus, and Neptune was another contributory factor to the discrepancies found in Gauss's time.

7. Gauss actually found that the value of the obliquity of the ecliptic deduced from solar observations made around the winter solstice was 10" less than that deduced from others made around the summer solstice. He interpreted this to mean that *at any time of year* solar observations would yield polar altitudes (or latitude) averaging 5" less than those from observations of circumpolar stars. From this he concluded that if all zenith distances were observed to be 2".5 smaller than they are in reality, the problem would be solved. See the extract from Gauss's letter of 1 February 1818 to Johann Elert Bode in the *Astronomisches Jahrbuch für 1818* (Berlin, 1815), 167-173.

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